

Limites

Exercice 1: $\lim_{x \rightarrow +\infty} \frac{\sin(x)}{\ln(x)} = ?$

Correction: Théorème d'encadrement :

$$0 \leq \left| \frac{\sin(x)}{\ln(x)} \right| \leq \frac{1}{\ln(x)} \xrightarrow{x \rightarrow +\infty} 0$$

Exercice 2: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin(x) - \cos(x)}{x - \frac{\pi}{4}} = ?$

Correction: $\frac{\sin(x) - \cos(x)}{x - \frac{\pi}{4}} = \frac{\sqrt{2} (\cos(\frac{\pi}{4}) \sin(x) - \sin(\frac{\pi}{4}) \cos(x))}{x - \frac{\pi}{4}}$
 $= \frac{\sqrt{2} \sin(x - \frac{\pi}{4})}{x - \frac{\pi}{4}}$

On pose $X = x - \frac{\pi}{4} \rightarrow 0$

on a: $\left[\sqrt{2} \frac{\sin(x)}{x} \rightarrow \sqrt{2} \right]$

Exercice 3: $\lim_{x \rightarrow -\infty} e^{-x} \ln(1+e^x)$

On pose $X = e^x \rightarrow 0$

Donc, $e^{-x} \ln(1+e^x) = \frac{\ln(1+X)}{X} \rightarrow 1$